Астрономия, математика, библиотеки Древней Греции

Ancient Greek Astronomy, Mathematics, Libraries

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Обзор тысячелетней истории древнегреческой астрономической и математической науки и достижений с точки зрения вклада в формирование современной науки и общества. Рассматриваются также достижения в области механики и астрономических расчетов. Выдающиеся научные открытия древности были бы невозможны без крупных греческих библиотек, существовавших в таких интеллектуальных центрах, как Афины, Александрия, Пергам, Родос, Эфес, Рим и Константинополь, а также финансировавшихся государством научных исследований в научных школах, таких как александрийская Библиотека и Мусейон – первый в мире университет. После падения Александрии часть научного наследия Древней Греции была унаследована средневековым Западом и Россией, в частности, на него опирались Колумб и Коперник.

The thousand-year history of ancient Greek astronomy and mathematics is reviewed, to illustrate the extent of its discoveries and the degree to which modern science and the modern world are indebted to it. Ancient Greek mechanical ingenuity and astronomical computing machinery are also discussed. This remarkable ancient history of scientific discovery would not have been possible without the growth of large Greek-language libraries in intellectual capitals like Athens, Alexandria, Pergamum, Rhodes, Ephesus, Rome and Constantinople, and the development of state-sponsored scientific scholarship in research centers like the Library and Museion of Alexandria, the world's first university. After the fall of Alexandria, some paths of transmission of ancient Greek scientific knowledge to the medieval West, Columbus, Copernicus and Russia are noted.

The Precursors

Before the rise of ancient Greece, the Egyptians studied the movement of sun, moon and stars to create a calendar and predict the annual life-giving floods of the Nile.[1] They also developed a practical knowledge of surveying and geometry that enabled them to build the pyramids.

The Babylonians devised a timekeeping system and mathematics based on the number 60 (conveniently divisible by 2, 3, 4, 5, 6, 10, 12, 15, 30). We still use it in measuring time and the arc angles of a circle (360^0). Their calendar, like ours, involved a mixture of the solar year of 365 days with lunar months (29½ days between repeated new moons), the months being divided into four seven-day weeks corresponding roughly to the Moon's four monthly phases. Twelve lunar months were 355 solar days and thirteen lunar months were 384 days, so the Babylonians reconciled the lunar and solar calendars by periodically inserting a 13th month into the solar calendar year. They also observed a cyclical pattern of eclipses of the Moon over 18 solar years (223 months) which allowed them predict them for astrological purposes.[2]

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1 Giorgio Abetti, *The History of Astronomy,* translated from the Italian by Betty Burr Abetti, London & New York: Abelard-Schuman, 1952, pp. 20-21. The Egyptians used a solar calendar of 365 earth rotations (observed as daily transits of the Sun across the sky). But the actual solar year, in which the Sun returns to the same place observed during its periodic eclipses against the background of the fixed stars, is within about 12 minutes of 365¼ days. This meant that the seasons, and the Egyptian New Year as marked by the annual flooding of the Nile in our August, rotated through a wandering calendrical year, occurring 1 day later each fourth year in the 365-day civil calendar. The flooding of the Nile was observed in 2782 B.C. to coincide with the appearance just before dawn (heliacal rising) of the brightest fixed star in the sky, Sothis (the "Dog Star' Sirius). This formed the basis of an astronomical cycle of 1460 heliacal risings of Sirius, or 1461 (4 x 365 + 1) wandering calendar years, after which the flooding of the Nile and the calendar New Year would again coincide exactly. Cf. Bradley E. Schaefer, "The heliacal rise of Sirius and ancient Egyptian chronology", *Journal for the History of Astronomy,* Vol. 32 Part 2 (May 2000), pp. 149-155.

2 Abetti, *History of Astronomy,* pp. 12-19.

The Minoan and Mycenaean Greeks designed multi-storey palaces and labyrinths using practical mathematical tools. In particular, the geometric shapes and proportions of Mycenaean monuments such as the Treasury of Atreus indicate early knowledge of the Pythagorean theorem.[3]

Thales of Miletus[4]

Θαλῆς ὁ Μῑλήσιος

Thales, who lived ca. 624 – ca. 546 B.C., was an Ionian Greek from the powerful city state of Miletus on the western coast of Asia Minor (now Turkey). He had travelled to study in Egypt, and became known as Greece's first scientific philosopher, and one of the Seven Sages.

In astronomy, Thales was the first Greek cosmologist. He thought the universe had originated in water, air evaporating from it and the solid Earth floating in it, with earthquakes being caused by agitation in the water. He was famed for having correctly predicted that there would be a solar eclipse in 585 B.C., although he probably only knew that one was about due to happen, rather than predicting the exact date. According to Plato, so absorbed was Thales in studying the heavens while out walking at night that he once fell into a ditch – the first known absent-minded professor.

In mathematics, Thales introduced the idea of a logical proof of theorems based upon deductive reasoning. Several of the basic theorems of geometry are associated with his name:

- *A circle is bisected by any diameter.*
- *The base angles of an isosceles triangle are equal.*
- *The angles between two intersecting straight lines are equal.*
- *Two triangles are congruent if they have two angles and one side equal.*

His most famous proof was that if a triangle is inscribed in a circle such that its longest side lies along the circle's diameter, then no matter where the third vertex of the triangle falls on the circle, the angle at that vertex is a right angle. Thales also knew how to use geometry to calculate the heights of pyramids and the distance of ships from shore.

Pythagoras of Samos[5]

Πυθαγόρας ὁ Σάμιος

Pythagoras lived ca. 570 - ca. 495 B.C. He was an Ionian philosopher who migrated with his followers from the Aegean island of Samos to the colonies of Magna Graecia (southern Italy). He discovered the tuning of musical instruments by the mathematical proportions of sizes of their vibrating elements, like string lengths in a violin or anvils in a blacksmith's shop.

In mathematics he is famous for deducing the general proof of the Pythagorean theorem, that in a right-angled triangle the area of the square on the longest side, or hypotenuse, equals the sum of the areas of the squares on the other two sides. Special cases of this theorem, such as the right triangle with sides in proportions 3:4:5, for which the squares on the sides have areas 32 + 42 = 52, had been known in Mycenaean design and decoration from the time of the Iliad.

In cosmology, Pythagoras was the first to propose that the Earth is spherical, not flat, that it might rotate, and that might not even be at the centre of the universe. The Sun or Central Fire was at the centre of things, and for the sake of symmetry the Pythagoreans hypothesized an unseen anti-Earth on the other side

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3 E. D. Chiotis, "New evidence on the advanced level of the Mycenaean mathematics", in *Archaeometry Issues in Greek Prehistory and Antiquity,* edited by Y. Bassiakos, E. Aloupi and Y. Facorellis, Athens: Hellenic Society for Archaeometry and Society of Messenean Archaeological Studies, 2001, pp. 801-810.

4 J. L. E. Dreyer, *A History of Astronomy from Thales to Kepler, formerly titled History of the Planetary Systems from Thales to Kepler,* 2nd ed., revised by W. H. Stahl, New York: Dover, 1953, pp. 11-13; *Astronomers, e*d. David Abbott, London: Frederick Muller/ Blond Educational, 1984 (The Biographical Dictionary of Scientists), "Thales of Miletus", pp. 151-152; *Biographical Dictionary of Mathematicians,* ed. Donald R. Franceschetti, 2 vols., New York: Marshall Cavendish, 1999, vol. 2, "Thales of Miletus", pp. 506-507; *Wikipedia, "*Thales of Miletus"; J.J. O'Connor and E.F. Robertson, "Thales of Miletus", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/ Biographies/Thales.html

5 *Biographical Dictionary of Mathematicians,* vol. 2, "Pythagoras", pp. 442-444; Dreyer, *History of Astronomy from Thales to Kepler,* pp. 35-52; Abetti, *History of Astronomy,* pp. 19-20; *Wikipedia, "*Pythagoras"; J.J. O'Connor and E.F. Robertson, "Pythagoras", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Pythagoras.html

of the Sun. Pythagoras believed in reincarnation; he thought that the souls of those who died were reborn elsewhere on the Earth, beyond the horizon.

Pythagoras was convinced that the proportions of numbers were the key to the universe. He held that the planets (wandering stars) moved against the background of the more distant fixed stars because they too were orbiting the Central Fire according to mathematical rules, such that their orbits resonated in proportions like tuned instruments, to produce a music of the spheres. It was he who began the long quest to analyse the mathematics of observed planetary motions.

Pythagoras or another Greek astronomer of his school, Philolaus, is said to have discovered that the bright morning and evening stars, alternately rising before the sun or setting after it, are the same bright planet, Venus, seen at different stages of its orbit around the Sun.

Pythagoras founded a notable school of thought, with his disciples carrying on his scientific and esoteric work in their master's name in the Greek colonies of southern Italy for at least two centuries after his death. His doctrine of the Central Fire may have influenced the astronomical ideas of Plato.[6]

Anaxagoras of Clazomenae[7]

Ἀναξαγόρας ὁ Κλαζομένιος

Anaxagoras, another Ionian philosopher, from Clazomenae in the Persian lands of Asia Minor, lived ca. 510 – ca. 428 B.C. He was the first thinker to bring scientific ideas to Athens, and was the tutor of the Athenian statesman Pericles.

Anaxagoras made his name as an observational and theoretical astronomer. He observed solar and lunar eclipses and explained them correctly in terms of orbits passing through planetary shadows. He also offered scientific explanations of rainbows and meteors, and investigated the great meteorite that fell at Aegospotamoi in Thrace in 467 B.C. around the time of the first recorded European observation of Halley's Comet.

In cosmology, Anaxagoras thought that the Sun was a sphere or plate of molten metal at least as large as the Peloponnese. Like Thales, he thought that the Earth was flat, floating not in water but in "strong air" (ether) which carried cosmic sound waves - the cause of earthquakes.

Meton of Athens[8]

Μέτων ὁ Ἀθηναῖος

Meton was the first native-born Athenian astronomer, living ca. 460 - ca. 400 B.C., and would have known Anaxagoras. He was recognisable enough as a public figure that he was even portrayed, carrying his astronomer's surveying instruments, in a walk-on part in Aristophanes' comedy *The Birds.*

The foundations of Meton's observatory are still visible behind the Pnyx, the hill of meeting of the Athenian people's Assembly.

From this location Meton studied the seasonal path, or ecliptic, of the Sun across the sky of Athens against the background of the fixed stars, and the timing of the summer and winter solstices and moon phases. He noted the differing lengths of the four seasons, which implied a changing speed of apparent solar motion along the ecliptic and, in modern terms, an elliptical, not circular, orbit of Earth about the Sun. His goal was to be able to predict accurately the new year, which in the Greek calendar of the time was celebrated at the first new moon after the summer solstice.

Meton's key observations in 432 B.C. established a 19-year cycle of recurring solar and lunar calendar events (19 solar years = 235 lunar months almost exactly), allowing not only the prediction of the new year but eclipse predictions.

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6 Dreyer, *History of Astronomy from Thales to Kepler,* pp. 53-86; Abetti, *History of Astronomy,* pp. 31-32. Plato's astronomical ideas as found in *The Republic* is discussed by D'Arcy Thompson, "Plato's Theory of the Planets", *The Observatory* (October 1904) and *The Classical Review* (August 1910), abridged by J.J. O'Connor and E.F. Robertson, "D'Arcy Thompson on Plato and Planets" (April 2007) in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Extras/Thompson_Plato.html .

7 Dreyer, *History of Astronomy from Thales to Kepler,* pp. 30-33; J.J. O'Connor and E.F. Robertson, "Anaxagoras of Clazomenae", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Anaxagoras.html; *Wikipedia, "*Anaxagoras".

8 Dreyer, *History of Astronomy from Thales to Kepler,* p. 93; Abetti, *History of Astronomy,* p. 34; *Wikipedia, "*Meton of Athens".

Eudoxus of Cnidus[9]

Εὔδοξος ὁ Κνίδιος

Eudoxus of Cnidus lived ca. 390 – ca. 337 B.C. He was a mathematician from Asia Minor who migrated to Athens, where he studied under Plato and taught Aristotle.

Eudoxus introduced the formal process of deducing proofs from a small number of geometric axioms. He specialized in the mathematical study of cones, spheres, and cylinders, but also began the study of quantities like square roots which are irrational, that is which cannot be expressed exactly as a ratio of two whole numbers. His theory of irrationals as lying between the rational numbers anticipated the 19th century development of the continuous real number system.

Plato set Eudoxus the problem of describing mathematically the observed planetary orbits around a spherical Earth. Eudoxus put each planet at the intersection of several celestial spheres with the same centre but different rotations (or homocentric spheres), to explain the observed variations in the planet's motion. But did not deal with the variations of a planet's brightness, which are now known to be due to its changing distance from the Earth.

As a practical astronomer, Eudoxus invented the astronomical globe, a kind of model of the sphere of fixed stars as seen by the gods from outside. This served not only as a tool to map the positions of the stars, but enabled Eudoxus from repeated observations to trace on his globe both the paths of the planets and the annual path of the noonday Sun in the sky among the fixed stars (the ecliptic). The ecliptic was inclined, as Meton had already observed, at an angle of about 23½^0 to the celestial equator, that latitude on the celestial sphere midway between the north and south celestial poles as defined by the axis of the Earth's daily rotation.

Callippus of Cyzicus[10]

Κάλλιππος ὁ Κυζικήνος

Callippus, who lived ca. 370 - ca. 300 B.C., was a Greek astronomer born just south of the Sea of Marmara, near Byzantium. In the epoch of Alexander's conquests, he studied under Eudoxus at Plato's Academy in Athens, and also at Aristotle's Lyceum.

Callippus fully mastered the system of homocentric spheres devised by his teacher Eudoxus at Plato's command to describe the motions of the Sun, Moon and planets, and extended it in order to account more closely for the observed motions of the celestial bodies.

> Callippus introduced a system of 34 spheres to explain the motions of the heavenly bodies. The Sun, Moon, Mercury, Venus and Mars each had five spheres while Jupiter and Saturn had four and the stars had one. This addition of six spheres over the system proposed by Eudoxus increased the accuracy of the theory while preserving the belief that the heavenly bodies had to possess motion based on the circle since that was the 'perfect' path.[11]

In particular, Callippus was trying in the case of the Sun's motion to explain the differing lengths of the four seasons discovered by Meton, which Eudoxus had ignored. This phenomenon implied a changing speed of apparent solar motion among the fixed stars, or in modern terms an elliptical, rather than circular, orbit of Earth about the Sun. But in the system of Callipus it required the introduction of additional interacting Earth-centred spheres to drag or accelerate the year-round motion of the noonday Sun against the background of the fixed stars along the ecliptic.

Callippus also established a more precise 76-year cycle of solar and lunar astronomical events that repeat, by taking the Metonic cycle of 19 years x 4, plus a leap day. The era of Callippus, when his very

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9 *Biographical Dictionary of Mathematicians,* vol. 1, "Eudoxus of Cnidus", pp. 177-179; *Astronomers*, "Eudoxus of Cnidus", pp. 50-51; Dreyer, *History of Astronomy from Thales to Kepler,* pp. 87-108; Abetti, *History of Astronomy,* pp. 32-34; J.J. O'Connor and E.F. Robertson, "Eudoxus of Cnidus", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Eudoxus.html ; *Wikipedia,* "Eudoxus of Cnidus".

10 Dreyer, *History of Astronomy from Thales to Kepler,* pp. 103-107; Abetti, *History of Astronomy,* pp. 34-35; *Wikipedia,* "Callippus".

11 J.J. O'Connor and E.F. Robertson, "Callipus of Cyzicus", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Callippus.html

precise eclipse prediction cycle began, started on 28 June 330 B.C. and remained in use throughout ancient times.

Euclid of Alexandria[12]

Εὐκλείδης ὁ Ἀλεξανδρεύς

The famous Euclid, who lived ca. 325 - ca. 270 B.C., worked as a mathematician and taught at the newly founded research centre and library of Alexandria in Hellenistic Egypt.

Euclid was the author of the standard textbook of geometry, the *Elements*, with step-by-step deductive proofs of theorems from axioms, still in use today after 23 centuries. For the most part he was a teacher compiling other geometers' work; the fifth book of the *Elements*, for example, derives largely from Eudoxus. But Euclid made notable advances of his own in number theory (prime numbers, unique prime factors of other numbers, greatest common divisors).

As well, Euclid wrote two important astronomical textbooks. His *Phaenomena*, on spherical astronomy, developed a system of celestial coordinates still in use. His *Optics* dealt with the theory of perspective of distant objects, moving or stationary, as seen by a moving or a stationary eye. In particular, one theorem in Euclid's Optics showed that

> when several objects move at unequal speed [and] the eye also moves in the same direction, [the objects] moving with the same speed as the eye will seem to stand still [against a more distant background], others moving more slowly [than the eye] will seem to move in the opposite direction, and others moving more quickly will seem to move ahead.[13]

This was the basis for the astronomical analysis of parallax, or the change of angular position and even direction of motion against the stellar background of relatively nearby objects like the Moon or a planet when viewed from different angles or places at the same or different times on the Earth, as reflecting the speed of motion and distance of the heavenly bodies themselves.[14]

Aristarchus of Samos[15]

Ἀρίσταρχος ὁ Σάμιος

The Ionian Greek astronomer Aristarchus of Samos lived ca. 310 – ca. 230 B.C. A rebel against accepted ideas, he is noteworthy in the history of astronomy for two great innovations.

Aristarchus reasoned that, by the same geometrical methods used to calculate the distance of a ship at sea, it should be possible to calculate the true distance from the Earth to the Sun and to the Moon. His treatise *On the Sizes and Distances of the Sun and the Moon* (Περὶ μεγεθῶν καὶ ἀποστημάτων ἡλίου καὶ σελήνης) was based on precise observations of their angular sizes, as seen in the moon's phases and in solar and lunar eclipses. His results for the Moon were not too bad by modern standards; but for the Sun, much too small, due to the observational limitations of naked-eye astronomy in estimating small angular variations in the Sun's apparent size.

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12 *Biographical Dictionary of Mathematicians,* vol. 1, "Euclid", pp. 173-176; Dreyer, *History of Astronomy from Thales to Kepler,* p. 88; Abetti, *History of Astronomy,* pp. 38; J.J. O'Connor and E.F. Robertson, "Euclid of Alexandria", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/ Biographies/Euclid.html; *Wikipedia, "*Euclid".

13 *The Optics of Euclid,* translated by Harry Edwin Burton, in *Journal of the Optical Society of America,* Vol. 35 No. 5 (May 1945), pp. 357-372, at p. 371.

14 The stars, by contrast, must be much further away than the Moon or the planets as they show no parallax when viewed at the same time from points far apart on the Earth's surface. However, with the development of the precision measurements of modern stellar astronomy the theory of parallax came into its own, since observations six months apart from opposite ends of the Earth's orbit do give a long enough base line, of twice the Earth's mean distance of 150 million km from the Sun, to allow the measurement of an angular displacement of the nearer stars and the direct calculation of their distance. We now know from such measurements the actual distance of most of the bright stars in the night sky, which varies from 4 to about 2500 light years. But for the ancient astronomers, the lack of an observable stellar parallax put all the stars at the same distance, on the sphere of fixed stars. Cf. *Wikipedia,* "Parallax".

15 *Astronomers*, "Aristarchos", pp. 12-13; Dreyer, *History of Astronomy from Thales to Kepler,* p.135-141; Abetti, *History of Astronomy,* pp. 37-38; *Wikipedia, "*Aristarchus of Samos"; J.J. O'Connor and E.F. Robertson, "Aristarchus of Samos", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/ Biographies/Aristarchus.html

More radical was Aristarchus's suggestion of a heliocentric theory of the solar system. It was known that the observed daily motion of the Sun across the sky was caused by the spherical Earth's daily rotation. But Aristarchus suggested that the observed annual solar path along the ecliptic against the background of the fixed stars could also be accounted for if the rotating Earth was going around the Sun. The book in which Aristarchus made the suggestion has not survived, but Archimedes reports that in it his fellow scientist hypothesized a universe in which Earth and the other planets do go around the Sun, and the fixed stars are other suns, very far away. Copernicus revolutionized planetary astronomy by returning to Aristarchus's model 1800 years later.

Archimedes of Syracuse[16]

Ἀρχιμήδης ὁ Συρακούσιος

Archimedes, recognized as the greatest scientist of Antiquity, lived ca. 287 – 212 B.C. He was the son of an astronomer of Magna Graecia, from Syracuse in Sicily, where he was born and died. However, he knew scientists from Alexandria and perhaps had studied there as well.

In mathematics Archimedes developed methods for dealing with irrational numbers such as square roots and π, the ratio of the circumference to the diameter of a circle, and calculating them to any desired degree of approximation. He explored converging geometric series and the properties of spheres, cylinders, conoids, and spirals. He invented exponential notation for very large numbers, not as we use it in terms of powers of two, ten or twelve, but in terms of powers of the myriad (10,000). To obtain some of his results he had to invent and use the infinitesimal calculus, which would only be rediscovered 1900 years later by Isaac Newton.

In physics Archimedes' outstanding contribution was *On floating bodies*, containing the theory of a force of buoyancy supporting a body immersed in water, equivalent to the volume of water that body displaced. Set the task of measuring the volume of a royal crown of known weight to determine if it was made of counterfeit metal, he realized as he sat in his bath that the water spilling over must be equivalent in volume to his own immersed body, instantly saw the solution to his problem and the whole theory of buoyancy in a nutshell, and ran naked through the streets of Syracuse crying Εὕρηκα! (I have found it!)

In civil and military engineering Archimedes came up with a host of practical inventions: the water screw, the odometer, leverage systems, block-and-tackle pulleys, catapults, heat rays to set enemy ships on fire and a mechanical jaw to crush them. His war machines delayed the Roman conquest of Syracuse by years, and when the city finally fell in 212 B.C. the Roman general was furious that a soldier against orders killed the elderly genius at work, instead of taking him captive.

Inheriting his father's interest in astronomy, Archimedes developed a mechanical planetarium to model the celestial motions around the Earth. But in his *Sand Reckoner* Archimedes also discussed Aristarchus's heliocentric theory and used its suggestions as to the distance of the fixed stars to calculate the volume of the universe, which he estimated at some 8 x 1063 grains of sand.[17]

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16 *Biographical Dictionary of Mathematicians,* vol. 1, "Archimedes", pp. 22-26; J.J. O'Connor and E.F. Robertson, "Archimedes of Syracuse", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Archimedes.html; *Wikipedia, "*Archimedes".

17 Archimedes had proven by the methods of calculus that the volume v of a sphere is $^4/_3\, \pi\, r^3$, where $^4/_3\, \pi$ equals approximately 4.18 and *r* is the radius of the sphere. In modern notation, taking a typical sand grain to have a diameter of half a millimeter and a radius of 0.25 mm, one spherical sand grain would have a volume of 6.48×10^{-11} m^3. Since spheres do not fill up a space completely but can only be packed into it to a maximum density of about 74% of the total containing volume, the rest being empty interstitial space, the effective volume contribution of each of a multitude of close-packed spherical sand grains in filling a container becomes about $^{100}/_{74} \times 6.48 \times 10^{-11}$ m^3 = 8.76×10^{-11} m^3. It is not clear if Archimedes took into account the packing fraction, but assuming he did so, his estimate of 8×10^{63} grains of sand filling the universe would correspond to a volume V contained within the outer sphere of fixed stars in Aristarchus's cosmology of about 7×10^{54} m^3, and a universe centred on the solar system of radius R = $\sqrt[3]{(^7/_{4.18} \times 10^{54}\ \text{m}^3)} \approx 1.19 \times 10^{18}$ m or about 126 light years. Given that 11 of the 20 brightest stars observable by the naked eye lie between 4 and 150 light years away (*Wikipedia,* "List of brightest stars"), the estimate of Aristarchus and Archimedes really is not bad as a median value for the size of the universe visible without telescopes in his day!

Eratosthenes of Cyrene[18]

Ἐρατοσθένης ὁ Κυρηναῖος

Eratosthenes, who came from Cyrene, a prosperous Greek colony on the Mediterranean coast of Libya, lived 276-194 B.C. He worked in Athens and then in Alexandria as an all-round scholar ("the second best at everything"): astronomer, mathematician, geographer, historian, poet and librarian. Archimedes counted him as his friend and intellectual equal.

As an astronomer, Eratosthenes tried to refine the work of Aristarchus on the distances from the Earth to the Sun and Moon and their sizes. Realising that the solar year, completed each summer solstice when the Sun returned to the same point high in the Egyptian sky as mapped against the background of the fixed stars, was about six hours longer than 365 days, Eratosthenes invented the Egyptian solar calendar of 365¼ days, with an extra leap day added every four years, which we still use today.

But as an astronomer Eratosthenes was most famous for the Syene experiment, in which he measured the spherical circumference $2\pi r$ and radius r of the Earth by ground-based observations of the length of the Sun's shadow, as cast by a vertical stake at high summer in two different locations. The locations selected for comparison were Alexandria on the Mediterranean and the Egyptian frontier post of Aswan or Syene on the upper Nile, roughly 5000 Greek stadia or 800 km south of Alexandria, where the Sun would be highest in the sky at approximately the same time. It was simple geometry to translate the length of the Sun's shadow at noon into the angular elevation of the Sun in the sky above the southern horizon, and at Syene, once a year, on the day of the solstice only, when the Sun reached its northernmost point in the sky, a vertical stake there cast practically no shadow and the Sun's reflection could be seen in the bottom of a deep well. This meant that Syene was on the Tropic of Cancer, the latitude on the Earth's surface where the Sun was directly overhead at the solstice, 90^0 above the horizon. Using the length of an identical stake's shadow in Alexandria, Eratosthenes was able to calculate the angular elevation of the Sun above the horizon at the solstice there, which turned out to be about 7^0 of arc less than in Syene. If a north-south distance of 5000 stadia corresponded to an angular difference of 7^0 in the Sun's elevation because of the Earth's curvature, and 7^0 of arc was about 1/50 of the Earth's circular measure of 360^0, then Eratosthenes had his calculation of the circumference and radius of the Earth. His results differed from the accepted modern values by only about 10%.

From this start Eratosthenes as a geographer proceeded to measure and map the extent of the known world from a prime meridian through Alexandria, using observations of the Sun to establish lines of longitude and latitude through different places on the globe. He also recognized that the variable solar elevation in the sky at different latitudes at the solstice or at the same latitude at different times of the year led to different levels of surface heating and a different climate. He was the first to correctly describe the Earth's five climate zones in astronomical terms and relate them to the $23½^0$ tilt of the spherical Earth's equator and axis of rotation in relation to the plane of the ecliptic, the annual path of the noonday Sun across the sky.

Between the latitude of Syene, or Tropic of Cancer, the northernmost point where the Sun at noon was observed directly overhead, and a corresponding latitude in the unknown lands south of the equator, the Tropic of Capricorn, the southernmost point where the Sun would be observed directly overhead, there lay the tropical lands, the hottest region of the Earth. North and south of the tropics lay the temperate zones, more favorable to civilization. The seasons in the Antipodes would be reversed from those in the North Temperate Zone. But traveling still further, beyond latitude $67½^0$ (90^0 -$23½^0$) north or south as measured from the equator, one would come to Frozen Lands (the Arctic or the Antarctic). Near the poles of the Earth's axis of rotation, though at opposite times of the year, in these polar regions for half of the year the Sun did not even rise above the horizon, but in the other half of the year it remained low in the sky for 24 hours, never setting but not providing enough heat to melt the ice.

[18] *Biographical Dictionary of Mathematicians,* vol. 1, "Eratosthenes", pp. 166-168; *Astronomers*, "Eratosthenes", pp. 49-50; Dreyer, *History of Astronomy from Thales to Kepler,* pp.174-176; J.J. O'Connor and E.F. Robertson, "Eratosthenes of Cyrene", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Eratosthenes.html; *Wikipedia,* "Eratosthenes".

As a mathematician Eratosthenes devised an algorithm (the Sieve of Eratosthenes) for finding ever larger prime numbers. He also solved the problem of doubling the volume of a cube, which involved calculating the cube roots of its sides.

Finally, as an historian and the librarian of Alexandria ca. 230-195 B.C. Eratosthenes compared histories written in various dating systems to establish the first general chronology of the ancient world, going back to the Trojan War in 1184 B.C.

Apollonius of Perga[19]
Ἀπολλώνιος ὁ Περγαῖος

Apollonius hailed from a Hellenistic town in Asia Minor near to the kingdom and great library of Pergamum, and lived ca. 240/230 - ca. 190/180 B.C. He studied and worked as a mathematician all his life, both in Pergamum and at the library of Alexandria in Egypt.

Apollonius was a pure mathematician who studied advanced three-dimensional geometry. His famous and definitive work on the mathematics of the conic sections (curves created by slicing a plane through a double cone at different angles) would lead in modern times to the generalized analytic geometry of the circle, the ellipse, the parabola and the hyperbola. See Figure 1.

Apollonius's significant contribution to astronomy was his suggestion that planetary orbits are not circular but eccentric, like the conic sections, because of a secondary (deferent) rotation of the planet for some unspecified reason around a moving point of reference on its primary circular orbit (epicycle). Thus was born the system of epicycles, which came to replace Eudoxus's system of homocentric spheres in ancient and medieval astronomy, and helped to explain the observed cases of retrograde motion, where a planet for a time appeared to turn back on its orbit.

Taking away the complex circular machinery of the epicycles needed to save the observed phenomena of planetary motion in an Earth-centred universe, it turned out that Apollonius was fundamentally right, when John Kepler and later modern astronomers proved that the orbits of the planets are ellipses, but with the Sun (rather than the Earth) at one focus, and comets from outside the solar system go around the Sun in one-time parabolic or hyperbolic orbits.

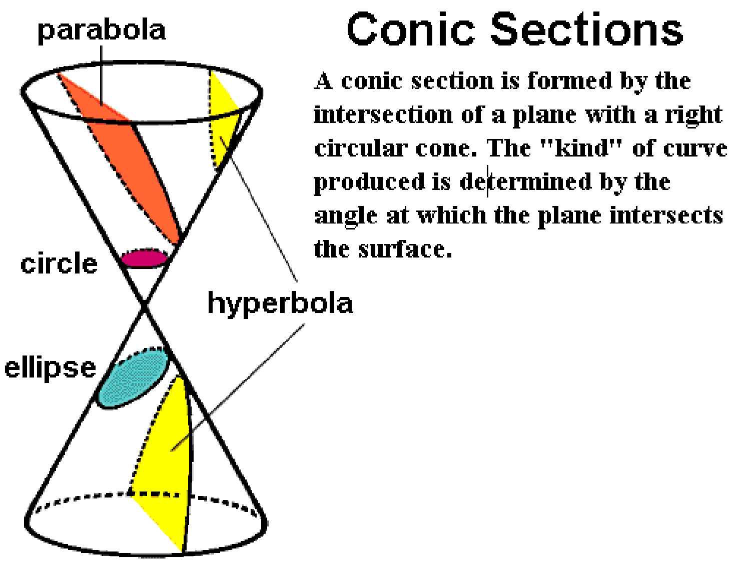

Figure 1. The Conic Sections[20]

[19] *Biographical Dictionary of Mathematicians,* vol. 1, "Apollonius of Perga", pp. 19-21; Dreyer, *History of Astronomy from Thales to Kepler*, pp. 151-156; J.J. O'Connor and E.F. Robertson, "Apollonius of Perga", in *MacTutor History of Mathematics Archive*, University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Apollonius.html ; *Wikipedia,* "Apollonius of Perga".

[20] Diagram after Jim Wilson, "The University of Georgia [College of Education]: Mathematics Education, EMAT 4600/6600", at http://wilson.coe.uga.edu/emt725/Conics Images/Conics.html as downloaded on 25 March 2018.

Hipparchus of Nicaea[21]

Ἵππαρχος ὁ Νικαιεύς

Hipparchus, considered the greatest observational astronomer of the ancient world, lived ca. 190 - ca. 120 B.C. He was born on the coast of Greek Asia Minor opposite Byzantium but made his career on the Greek Aegean island of Rhodes, a rival in learning of Alexandria and Pergamum.

As a mathematician, Hipparchus was the father of trigonometry (which he based on a study of the chord lengths subtended by different angles in a circle).

As an astronomer, he developed a detailed geocentric theory of the orbits, size, and distances of the Moon and Sun. He also discovered the precession of the Earth's rotation, a slow circular wobble like a top over 25,800 years, causing the north pole star to move.

Hipparchus was the inventor of the astrolabe and other portable astronomical instruments used for Sun sightings, and he improved Eratosthenes' geography by making star as well as Sun sightings of latitude and longitude. He created the first scientific star catalogue of the night sky, including star brightnesses (magnitudes) and mapping out the classical constellations.

The Antikythera Mechanism[22]

The ancient Greeks, as we saw with Archimedes' planetarium, were quite adept at constructing mechanical devices to represent or compute astronomical phenomena. Most of these were lost, but an exception is the Antikythera mechanism (Figure 2), discovered in 1902 in an ancient shipwreck off the island of Antikythera between the Peloponnese and Crete.

The ship went down around 87 B.C and it appears the mechanism found in it was a state-of-the-art portable computer of that time for a master mariner's use. Detailed study has shown it to be a complex instrument of about thirty gears for computing solar and lunar phenomena, including the 76-year cycle of Sun and Moon positions and tides discovered by Callippus.

Gear-based technology as sophisticated as this did not reappear until the large clocks and mechanical automata built in Western Europe between the 15th and the 18th centuries. In terms of computing strategies, the Antikythera mechanism was as sophisticated as the mechanical difference engines used in Victorian England to perform arithmetical computations, which were the direct forerunners of the modern electronic computer.

Figure 2. The Antikythera Mechanism

[21] *Biographical Dictionary of Mathematicians,* vol. 1, "Hipparchus", pp. 279-281; *Astronomers*, "Hipparchus", pp. 75-76; Dreyer, *History of Astronomy from Thales to Kepler,* pp. 160-169, 183, 193, 203; Abetti, *History of Astronomy,* pp. 40-42; J.J. O'Connor and E.F. Robertson, "Hipparchus of Rhodes", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Hipparchus.html; *Wikipedia,* "Hipparchus".

[22] "The Antikythera mechanism is a 2,100-year-old computer", posted on 5 January 2018 by the historical linguist vallance22 in his blog on Minoan Linear A, Linear B, Knossos & Mycenae in connection with arguments that an older Minoan computing device may have been found. Cf. https://linearbknossosmycenae.com/2018/01/05/the-antikythera-mechanism-is-a-2100-year-old-computer/ Compare *Wikipedia,* "Antikythera mechanism", as at 25 March 2018.

Ptolemy of Alexandria[23]

Κλαύδιος Πτολεμαῖος

Claudius Ptolemy, who lived and worked ca. 100 - ca. 170 A.D. in Alexandria, was a Greco-Roman-Egyptian astronomer, mathematician, geographer, music theorist and astrologer.

His *Μαθηματικὴ Σύνταξις (Mathematical Treatise)* of ca. 150 A.D. on astronomy was later called *Ἡ Μεγάλη Σύνταξις or Ἡ Μεγίστη Σύνταξις*, a title translated into Arabic and thence into European languages as Almagest.

Ptolemy's great book, copied countless times, canonized the geocentric model of the universe with epicyclic motion of the Sun, Moon and five naked-eye planets (Mercury, Venus, Mars, Jupiter, Saturn) into an astronomical system unquestioned throughout the Middle Ages.

He also adjusted and extended Hipparchus's star catalogue and geography.

Diophantus of Alexandria[24]

Διόφαντος ὁ Ἀλεξανδρεύς

Diophantus was a mathematician of great originality who lived and worked in Alexandria a century after Ptolemy, ca. 201/215 - ca. 285/299 A.D. Greek mathematical methods until then had been heavily dependent on geometry, but Diophantus took a different approach. His *Arithmetica* was the major ancient Greek treatise on algebra and the solution of mathematical equations.

Diophantus was an innovator inasmuch as he admitted fractional as well as whole number (integer) solutions to mathematical problems. He also studied approximate equalities, as well as indeterminate equations which had multiple possible integer solutions.

In another innovation, Diophantus graphed the values of algebraic functions on a grid with two axes, corresponding to what we would now call the independent variable (x) and the dependent variable (y). However, he did so only in the first quadrant, no negative values being admitted.

His new mathematics prepared the way, after a hiatus of 14 centuries, for the development of both Cartesian analytical geometry and Newtonian orbital mechanics. Essentially, Diophantus provided the mathematical notation which Kepler and Newton would use in developing the equations for the heliocentric motion of the planets.

Pappus of Alexandria[25]

Πάππος ὁ Ἀλεξανδρεύς

Pappus of Alexandria, who lived ca. 290 - ca 350 A.D., was a great compiler of mathematical knowledge. His *Mathematical Collections* in eight books summarized and extended the work of most his predecessors. Often Pappus offers a valuable introduction to the historical development of a mathematical topic, mentioning authors otherwise lost.

Pappus's special interest was the study of solid geometry: the properties of spheres, spiral shells, and the five regular solids described by Plato (tetrahedrons, cubes, octahedrons, dodecahedrons, icosahedrons – there are no others).

By this time Ptolemy's treatise had become the standard great compendium of astronomy. Pappus wrote a commentary on it in which he mentioned himself having observed the solar eclipse of 18 October

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23 *Biographical Dictionary of Mathematicians,* vol. 2, "Ptolemy", pp. 439-441; *Astronomers*, "Ptolemy (Claudius Ptolemaeus)", pp. 130-131; Dreyer, *History of Astronomy from Thales to Kepler,* pp. 191-206; J.J. O'Connor and E.F. Robertson, "Claudius Ptolemy", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Ptolemy.html; *Wikipedia,* "Ptolemy", "Almagest" and "Hipparchus".

24 *Biographical Dictionary of Mathematicians,* vol. 1, "Diophantus", pp. 157-159; J.J. O'Connor and E.F. Robertson, "Diophantus of Alexandria", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Diophantus.html; *Wikipedia,* "Diophantus".

25 Serafina Cuomo, *Pappus of Alexandria and the Mathematics of Late Antiquity,* Cambridge: Cambridge University Press, 2000; *Wikipedia,* "Pappus of Alexandria"; J.J. O'Connor and E.F. Robertson, "Pappus of Alexandria", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Pappus.html

320 in Alexandria. He also commented upon and preserved the Little *Astronomy*, selected astronomical works by such older authors as Aristarchus and Eratosthenes.

Hypatia of Alexandria[26]

Ὑπατία ἡ Ἀλεξανδρίς

After so many men, at last a significant woman! Hypatia, the daughter of a minor Alexandrian astronomer, Theon II, surpassed her father's reputation for learning. Born somewhere between A.D. 350 and 370 into the elite Alexandrian intellectual upper class, she acquired a considerable reputation as a pagan neoPlatonist philosopher and teacher in an age when conversion to Christianity had become the norm.

But Hypatia was also important in the history of Greek science as a mathematician and an astronomer. In particular, she was a great editor and educator in these fields. She revised the classics to bring them up to date – clarifying proofs, adding commentary in Ptolemy, Euclid, Diophantus - and even wrote a popularized version of the difficult treatise on conic sections by Apollonius of Perga.

Hypatia's *Astronomical Canon*, comprising her calculations of the equinox of 414-415 to help the Roman governor of Egypt resolve a dispute about the dates of Passover and Easter, was her downfall, leading to her death in 415 at the hands of a mob of ignorant Christian monks – all males - who disagreed with her evidence-based conclusions.

Paradoxically, Hypatia's death is thought by some to have helped inspire the medieval Christian legend of the martyred St. Catherine of Alexandria. Since the 19th century she has come to be regarded as a martyr of philosophy, and more recently she has become a feminist icon. A leading contemporary journal of feminist philosophy bears her name, though chiefly discussing topics of racial and gender oppression which she never thought of.

Ancient Libraries and Research Centres[27]

The long history of ancient Greek mathematical and astronomical scholarship comes to an end in the fifth century A.D. But an important part of that history was how that scholarship was conserved and passed on down the thousand years from Thales to Hypatia.

The answer was the development of another great Greek innovation, the research library. Already the Academy in Athens had significant manuscript collections in the time of Plato and Aristotle. After the conquests of Alexander the Great, works of literature and learning were systematically collected under royal or private patronage in other major libraries across the Hellenistic world.

The most comprehensive Greek library collections were to be found at the Library of Alexandria in Egypt, the Library of Pergamum in Asia Minor, the Royal Library of Antioch, and from the 4th century A.D. the Byzantine Imperial Library of Constantinople. Other notable collections were found in Athens, in Rhodes, in Ephesus, and in the Forum of Trajan in Rome (the Bibliotheca Ulpiana, with separate branches for Greek and for Latin works).

The Library and Museion of Alexandria[28]

The port city of Alexandria in Egypt had been founded by Alexander the Great around 331 B.C. It soon became the capital of the Greek successor dynasty of the Ptolemies and of the intellectual life of the Hellenistic world, with a great library especially rich in holdings for researchers in mathematics and astronomy.

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[26] Edward J. Watts, *Hypatia: The Life and Legend of an Ancient Philosopher,* Oxford: Oxford University Press, 2017; J.J. O'Connor and E.F. Robertson, "Hypatia of Alexandria", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Hypatia.html; *Wikipedia*, "Hypatia".

[27] Lionel Casson, *Libraries in the Ancient World,* New Haven: Yale University Press, 2002; *Ancient Libraries,* ed. Jason König, Katerina Oikonomopoulou and Greg Woolf, Cambridge: Cambridge University Press, 2013.

[28] Mostafa El-Abbadi, *The Life and Fate of the Ancient Library of Alexandria,* rev. 2nd ed., Paris: UNESCO, 1992; *The Library of Alexandria: Centre of Learning in the Ancient World,* ed. Roy M. Macleod, London & New York: I.B. Tauris, 2000; James Hannam, "The Foundation and Loss of the Royal and Serapeum Libraries of Alexandria", 2003, last modified 2011, under the tab "The Great Library of Alexandria" in his Internet collection *Bede's Library: Articles and Essays on History, Science, Religion and Philosophy* at http://bede.org.uk/Library2.htm

Alexandria's library was organized under royal patronage around 290 B.C. and in its first centuries was directed by distinguished scholars. For example, its fifth head librarian, for about 35 years ca. 230-195 B.C., was the multitalented astronomer, geographer and historian Eratosthenes. The Museion (place of the Muses), attached to the library, was not as at Pergamum a museum in the modern sense, a collection of artifacts, but rather a research and teaching centre, a kind of university. In Alexandria's Museion a thousand scholars had free room and board at state expense.

The library's collections grew rapidly thanks to a sizeable staff of scribes and a deliberate policy, supported by the state, of copying all books available: from Rhodes or from Athens by copying expeditions, from ships that stopped in the busy port of Alexandria by royal command. The collections were copied on papyrus scrolls and indexed in an extensive catalogue.

The ancient Library of Alexandria endured in various forms for about nine centuries. The original library and Museion are said to have been damaged or even destroyed by Roman armies under Julius Caesar or in the suppression of the rebellion of Zenobia of Palmyra ca. 270 A.D. The library collections which survived were relocated to the Serapeum (temple of Serapis) ca. 200 A.D. but were confiscated in part and perhaps sent to the new Imperial Library at Constantinople in the 360s, not long before the Serapeum was destroyed as a pagan shrine by Alexandria's Christian bishop in the 390s. But the Museion may have lasted as a centre of study beyond the murder of Hypatia by Christian fanatics in 415, until the Persian and Arab invasions of the 600s.

The tale that the remains of the great library were burned by the Muslim Arab conquerors of Egypt in 641 is probably apocryphal. But in the early Middle Ages, as a result of earthquakes and flooding, the site of the ancient royal palace and Library sank beneath the waves of the city's harbour. After an international financing campaign, the modern Library of Alexandria, a world knowledge centre for Africa and the Mediterranean, opened next to the harbour in 2002.

The Transmission of the Heritage of Greek Science[29]

What happened to the books? Many had been lost over the course of the centuries, not only because of war or confiscation, but because older works of mathematics or astronomy went out of fashion with the publication of new, encyclopedic compendia by scholars like Euclid, Ptolemy or Pappus. But there were several channels of transmission to the rest of Europe for what remained.

In the 6th century the scholar Boethius in Ravenna and monks in southern Italy translated into Latin some of the Greek manuscripts from Constantinople, including basic textbooks of arithmetic, geometry and the theory of music which were passed on to medieval western Europe.[30] Astronomy for timekeeping was revived by the English scholar monk Bede the Venerable, ca. 673-735.[31]

The Muslim conquerors translated the most important works of ancient Greek philosophy, including mathematics and astronomy, into Arabic in Baghdad or Cairo, and these were later passed on through Muslim Spain to western Europe. An early example of this transmission was the mathematical and astronomical knowledge brought back from Spain in the 10th century by the Frankish scholar and teacher Gerbert of Aurillac (later Pope Sylvester II).[32] But the great flow into medieval western Europe of translations of ancient Greek scientific works took place between 1100 and 1300. In particular, the study of astronomy was revived in the new European universities when Ptolemy of Alexandria's work became available in translations made from the original Greek in Sicily in 1160 and from its Arabic incarnation, the Almagest, in Spain in 1175.

Some rescued ancient works from the Libraries of Alexandria and Pergamum, preserved in copies in the Imperial Library of Constantinople and surviving its purported destruction by the 4th Crusade in 1204,

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29 David C. Lindberg, "The Transmission of Greek and Arabic Learning to the West", chapter 2 of *Science in the Middle Ages,* Chicago: University of Chicago Press, 1978, pp. 52-90.

30 J.J. O'Connor and E.F. Robertson, "Anicius Manlius Severinus Boethius", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, http://www-history.mcs.st-andrews.ac.uk/Biographies/Boethius.html ; *Wikipedia,* "Boethius"; James J. O'Donnell, *Cassiodorus,* Berkeley: University of California Press, 1979, reproduced 1995 on the Internet at http://faculty.georgetown.edu/jod/cassiodorus.html (see especially chapter 6); *Wikipedia,* "Cassiodorus".

31 *Science in the Middle Ages,* chapter 1, p. 34.

32 J.J. O'Connor and E.F. Robertson, "Gerbert of Aurillac", in *MacTutor History of Mathematics Archive,* University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Gerbert.html

came to Italy in the Renaissance, ca. 1350-1450, during a movement for church reunion between the Eastern Orthodox and Roman Catholics. On the other hand, despite the early conversion of Russia to Orthodoxy in 988, there was no parallel movement to copy or teach the ancient Greek scientific knowledge from Constantinople in Russia until the late 17th century, and even then this attempt was made by Westernizers.[33]

But despite the conquest of Constantinople by the Ottoman Turks in 1453, ancient Greek works hidden in lesser libraries there were still being discovered by scholars in the 20th century. The Archimedes Palimpsest, a partially erased 10th-century parchment manuscript of lost works by the great scientist and other ancient Greek authors, overwritten by a 13th century book of prayers, was noticed in the 1840s, catalogued in 1899, studied by the Danish scholar Heiberg in 1906 but stolen in the 1920s. It resurfaced at auction in 1998 and was published in 2008 after full recovery of the underlying scientific text through X-ray, ultraviolet, visible light and infrared imaging.[34]

Ancient Greek Science and the Birth of the Modern World

In 1492 Columbus knew from Eratosthenes that the world was round and its size. But he had seen a recent Italian map that underestimated Earth's circumference by a quarter. China and India were much further away than he told his Spanish royal backers. As discovered by Balboa in 1513 and by Magellan in 1520, a whole New World for conquest and the Pacific Ocean lay between.[35]

The Polish astronomer Copernicus knew of the heliocentric system of Aristarchus of Samos as reported by Archimedes, and used it to explain the heavens. His publication of De *revolutionibus orbium coelestium* in 1543, the year of his death, together with Kepler's mathematical analysis in the early 1600s of the orbits of Earth and the other planets as ellipses with the Sun at one focus, revolutionized astronomy. They led to Newton's inverse-square law of gravitational attraction that governs the orbits of rockets and planets, and ultimately to the conquest of space.[36]

To the Moon

All the ancient Greek astronomers and mathematicians have features on the face of Earth's satellite named after them. To see Archimedes or Hipparchus or Eratosthenes or even Hypatia, just look at the full moon with a good pair of binoculars or a small telescope and a lunar map.[37]

[33] Efthymios Nicolaidis, *Science and Eastern Orthodoxy: From the Greek Fathers to the Age of Globalization*, translated by Susan Emanuel, Baltimore: Johns Hopkins University Press, 2011, chapter 11, "Toward Russia: The Slavo-Greco-Latin Academy and the Patriarchate of Jerusalem".

[34] "Big Literary Find in Constantinople: Savant Discovers Books by Archimedes, Copied about 900 A.D.", *New York Times,* 16 July 1907; Reviel Netz and William Noel, *The Archimedes Codex: Revealing the Secrets of the World's Greatest Palimpsest,* London: Weidenfeld & Nicolson, 2007; *Wikipedia,* "Archimedes Palimpsest".

[35] *Wikipedia,* "Christopher Columbus", "Vasco Núñez de Balboa" and "Ferdinand Magellan".

[36] J.J. O'Connor and E.F. Robertson, "Nicolaus Copernicus", in *MacTutor History of Mathematics Archive*, University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/Biographies/Copernicus.html; *Wikipedia,* "Nicolaus Copernicus", "Johannes Kepler" and "Isaac Newton".

[37] For example, Don Spain, *The six-inch lunar atlas: a pocket field guide,* New York: Springer, 2009.

Bibliography

1. Abetti, Giorgio. The History of Astronomy, translated from the Italian by Betty Burr Abetti, London & New York: Abelard-Schuman, 1952
2. Astronomers, ed. David Abbott, London: Frederick Muller/ Blond Educational, 1984 (The Biographical Dictionary of Scientists).
3. Articles on Aristarchos (pp. 12-13), Eratosthenes (pp. 49-50), Eudoxus of Cnidus (pp. 50-51), Hipparchus (pp. 75-76), Ptolemy (Claudius Ptolemaeus, pp. 130-131), and Thales of Miletus (pp. 151-152).
4. “Big Literary Find in Constantinople: Savant Discovers Books by Archimedes, Copied about 900 A.D.”, New York Times, 16 July 1907.
5. Biographical Dictionary of Mathematicians, ed. Donald R. Franceschetti, 2 vols., New York: Marshall Cavendish, 1999.
6. Articles on Apollonius of Perga (vol. 1, pp. 19-21), Archimedes (vol. 1, pp. 22-26), Diophantus (vol. 1, pp. 157-159), Eratosthenes of Cyrene (vol. 1. pp. 166-168), Euclid (vol. 1, pp. 173-176), Eudoxus of Cnidus (vol. 1, pp. 177-179), Hipparchus (vol. 1, pp. 279-281), Ptolemy (vol. 2, pp. 439-441), Pythagoras (vol. 2, pp. 442-444), and Thales of Miletus (vol. 2, pp. 506-507).
7. Casson, Lionel. Libraries in the Ancient World, New Haven: Yale University Press, 2002; Ancient Libraries, ed. Jason König, Katerina Oikonomopoulou and Greg Woolf, Cambridge: Cambridge University Press, 2013.
8. Chiotis, E. D. “New evidence on the advanced level of the Mycenaean mathematics”, in Archaeometry Issues in Greek Prehistory and Antiquity, edited by Y. Bassiakos, E. Aloupi and Y. Facorellis, Athens: Hellenic Society for Archaeometry and Society of Messenean Archaeological Studies, 2001, pp. 801-810.
9. Cuomo, Serafina. Pappus of Alexandria and the Mathematics of Late Antiquity, Cambridge: Cambridge University Press, 2000.
10. Dreyer, J. L. E. A History of Astronomy from Thales to Kepler, formerly titled History of the Planetary Systems from Thales to Kepler, 2nd ed., revised by W. H. Stahl, New York: Dover, 1953.
11. El-Abbadi, Mostafa. The Life and Fate of the Ancient Library of Alexandria, rev. 2nd ed., Paris: UNESCO, 1992
12. Euclid of Alexandria, The Optics of Euclid, translated by Harry Edwin Burton, in Journal of the Optical Society of America, Vol. 35 No. 5 (May 1945), pp. 357-372.
13. Hannam, James. “The Foundation and Loss of the Royal and Serapeum Libraries of Alexandria”, 2003, last modified 2011, under the tab “The Great Library of Alexandria” in his Internet collection Bede’s Library: Articles and Essays on History, Science, Religion and Philosophy at http://bede.org.uk/Library2.htm .
14. Heath, T. L. A History of Greek Mathematics, 1921 ed., 2 vol., republished Oxford: 1965.
15. Heath, T. L. Aristarchus of Samos, the ancient Copernicus: A history of Greek astronomy to Aristarchus, together with Aristarchus's Treatise on the sizes and distances of the sun and moon, 1913 ed., republished Oxford: 1966.
16. MacTutor History of Mathematics Archive, edited by J.J. O'Connor and E.F. Robertson, University of St. Andrews, Scotland, at http://www-history.mcs.st-andrews.ac.uk/ .
17. Articles on Anaxagoras of Clazomenae, Anicius Manlius Severinus Boethius, Apollonius of Perga, Archimedes of Syracuse, Aristarchus of Samos, Callipus of Cyzicus, Claudius Ptolemy, Diophantus of Alexandria, Eratosthenes of Cyrene, Euclid of Alexandria, Eudoxus of Cnidus, Gerbert of Aurillac, Hipparchus of Rhodes, Hypatia of Alexandria, Nicolaus Copernicus, Plato and Planets (D'Arcy Thompson), Pythagoras, Thales of Miletus.
18. Netz, Reviel and Noel, William. The Archimedes Codex: Revealing the Secrets of the World’s Greatest Palimpsest, London: Weidenfeld & Nicolson, 2007.
19. Neugebauer, Otto. History of Ancient Mathematical Astronomy. Berlin: Springer, 1975.
20. Nicolaidis, Efthymios. Science and Eastern Orthodoxy: From the Greek Fathers to the Age of Globalization, translated by Susan Emanuel, Baltimore: Johns Hopkins University Press, 2011.
21. O’Donnell, James J. Cassiodorus, Berkeley: University of California Press, 1979, reproduced 1995 on the Internet at http://faculty.georgetown.edu/jod/cassiodorus.html.
22. Pape, W. Benseler, G.E. Wörterbuch der griechischen Eigennamen: Dictionary of Greek proper names, 3rd ed. Braunschweig: 1884, at https://archive.org/details/wrterbuchdergr01papeuoft .
23. Schaefer, Bradley E. “The heliacal rise of Sirius and ancient Egyptian chronology”, Journal for the History of Astronomy, Vol. 32 Part 2 (May 2000), pp. 149-155.
24. Science in the Middle Ages, ed. David C. Lindberg, Chicago: University of Chicago Press, 1978. See especially Lindberg’s own chapter 2, “The Transmission of Greek and Arabic Learning to the West”, pp. 52-90.
25. Spain, Don. The six-inch lunar atlas: a pocket field guide, New York: Springer, 2009.
26. “The Antikythera mechanism is a 2,100-year-old computer”, posted on 5 January 2018 by the historical linguist vallance22 in his blog on Minoan Linear A, Linear B, Knossos & Mycenae, https://linearbknossosmycenae.com/2018/01/05/the-antikythera-mechanism-is-a-2100-year-old-computer/
27. The Library of Alexandria: Centre of Learning in the Ancient World, ed. Roy M. Macleod, London & New York: I.B. Tauris, 2000.
28. Thompson, D’Arcy. “Plato’s Theory of the Planets”, The Observatory (October 1904) and The Classical Review (August 1910), abridged by J.J. O'Connor and E.F. Robertson, “D'Arcy Thompson on Plato and Planets” (April 2007) in MacTutor History of Mathematics Archive, at http://www-history.mcs.st-andrews.ac.uk/Extras/Thompson_Plato.html.

29. Watts, Edward J. Hypatia: The Life and Legend of an Ancient Philosopher, Oxford: Oxford University Press, 2017.
30. Wikipedia articles as consulted online on dates between 25 March 2018 and 30 April 2018: Almagest, Anaxagoras, Ancient Greek Astronomy, Antikythera Mechanism, Apollonius of Perga, Archimedes, Archimedes Palimpsest, Aristarchus of Samos, Boethius, Callippus, Cassiodorus, Christopher Columbus, Diophantus, Eratosthenes, Euclid, Eudoxus of Cnidus, Ferdinand Magellan, Greek Mathematics, Hipparchus, Hypatia, Isaac Newton, Johannes Kepler, Library of Alexandria, Libraries in the Ancient World, List of Brightest Stars, Meton of Athens, Nicolaus Copernicus, Pappus of Alexandria, Parallax, Ptolemy, Pythagoras, Thales of Miletus, Vasco Núñez de Balboa.